LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2024**

UMT 5502 – LINEAR ALGEBRA

Date: 11-11-2024 Dept. No. Max. : 100 Marks
Time: 09:00 am-12:00 pm

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**Answer the following**
a)	Define a Subspace of a vector space.
b)	What is an orthonormal set?
c)	When a linear transformation is said to be regular?
d)	Define Matrix of a Linear Transformation
e)	Define Unitary transformation
2.	**Fill in the blanks**
a)	If $dim_F V = m then dim_F Hom(V, V)$ is ____
b)	If Vis a vector space over F, then Hom (V, F) is called as ____
c)	If T ∈ A(V), then VT, is defined by VT ={vT / v ∈ V} is called as ____ of T.
d)	The linear transformations $S, T \in A(V)$ are said to be similar if there exists an invertible element $C \in A(V)$ such that it satisfies the relation ________.
e)	T ∈ A(V) is unitary if and only if TT* = ______
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	Choose the correct answer from the following
a)	The vectors $v_1 =¿$ (1, 1, 0), v_2 = (1, 0, 1), and $v_{3=¿¿}$ (0, 1, 1) in R^3, then a) the vectors are Linearly Dependent b) the vectors are Linearly Independent c) $v_1$is Linear combination of v_2 d)$v_2 is a Linear combination of v_1$
b)	The norm of the Vector (2, 0, -1) is a. 5 b. -5 c. $\sqrt{5}$ d. None of these
c)	If V is finite-dimensional over F then for S, T ∈ A (V) a) r(ST)≤ r(T) b) r(ST) = r(T) c) r(ST) > r(T) d) None of these
d)	If $T \in A(V)$ and if $dim_F V = n$, then T can have at most _________ distinct characteristic roots in F. a) n b) n-1 c) n+1 d) one
e)	The Characteristic roots of a Skew-Hermitian matrix are a) only zero b) Only Real c) either Real or Imaginary d) either Zero Or purely Imaginary
4.	State True or False
a)	Any subset of a Linearly Independent set is Linearly dependent.
b)	$w^{\perp}$ is a subspace of the vector space V.
c)	If V is finite-dimensional over F, then T∈ A(V) is singular if and only if there exists a v ≠0 in V such that v T = 0.
d)	If $T A(V)$ has all its characteristic roots in F, then that there is no basis of Vin which the matrix of T is

	triangular.
e)	T E A(V) is called self-adjoint or Hermitian if T*= T.

SECTION B - K3 (CO2)	
Answer any TWO of the following in 100 words each.	**(2 x 10 = 20)**
5.	If V is a finite dimensional vectors space and $v_1, \ldots.. v_r$ is a Linearly Independent subset of V then it can be extended to form a basis of V.
6.	Deduce the proof of Schwarz inequality.
7.	If V is finite-dimensional over F, then prove that $T \in A(V)$ is singular if and only if there exists $a\, v \neq 0$ in V such that $vT = 0$.
8.	Let V be a set of all polynomials in x of degree ≤ 3 over F. Define D on V as: $(\beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3) D = \beta_1 + 2\beta_2 + 3\beta_3 x^2$.Determine the matrix of D.

SECTION C – K4 (CO3)	
Answer any TWO of the following in 100 words each.	**(2 x 10 = 20)**
9.	Show that the vectors v_1 = (0, 1, – 2), v_2 = (1, – 1, 1), v_3 =(1, 2, 1) are Linearly Independent in R^3
10.	If V is a finite-dimensional inner product space and if $W$is a subspace of V, then prove that $V = W + W^{\perp}$. More particularly, V is the direct sum of W and $W^{\perp}$.
11.	If $\lambda \epsilon F$ is a characteristic root of $\in A(V)$, then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$.
12.	Prove that if $T \epsilon A(V)$ then $T^{¿} \epsilon A(V)$. Moreover, for all $S, T \epsilon A(V)$ and all $\lambda \epsilon F$, show that 1. $(S+T)^{¿} = S^{¿} + T^{¿}$ 2. $(ST)^{¿} = T^{¿} S^{¿}$

SECTION D – K5 (CO4)	
Answer any ONE of the following in 250 words	**(1 x 20 = 20)**
13.	If V is finite-dimensional and if $W$is a subspace of V, then $Deduce\ that\ dim\, V/W = dim\, V - dim\, W$.
14.	If V is a finite-dimensional inner product space and $v_1, \ldots.. v_n$ is a basis of V then Construct an orthonormal basis for V by Gram-Schmidt orthogonalization process

SECTION E – K6 (CO5)	
Answer any ONE of the following in 250 words	**(1 x 20 = 20)**
15.	Prove that if $T \epsilon A(V)$ has all its characteristic roots in F, then there is a basis of V in which the matrix of T is triangular.
16.	a) If (vT, vT) = (v, v) for all v in V then Justify that T is unitary. (10 Marks) b) Deduce that "The linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V". (10 Marks)

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